ON SPECKLE DIFFRACTION INTERFEROMETRY FOR MEASURING WHOLE FIELD DISPLACEMENTS AND STRAINS

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AND STRAINS

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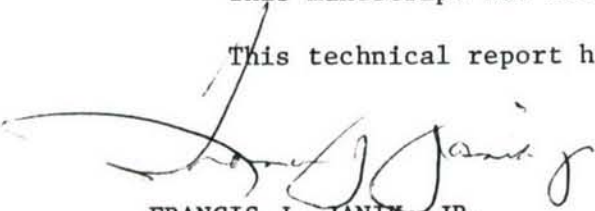
FRANK D. ADAMS

and

GENE E. MADDUX

FOREWORD

This report is the result of an in-house effort conducted under Project 1467, "Structural Analysis Methods", Task 146702, "Thermo-elastic Structural Analysis Methods". Experimental work was performed in the Air Force Flight Dynamics Laboratory Photomechanics Facility.

The work reported herein was conducted from June 1972 to June 1973 by Dr. Frank D. Adams, Physicist, and Mr. Gene E. Maddux, Aerospace Engineer, both in the Analysis Group, Structures Division, Air Force Flight Dynamics Laboratory. Mr. Robert M. Bader, AFFDL(FBR), is the Technical Manager of the Analysis Group.

This manuscript was released by the authors in August 1973.

This technical report has been reviewed and approved.

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Chief, Solid Mechanics Branch
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ABSTRACT

The procedure for using laser speckle diffraction inter-
ferometry to measure small displacements and strains is described.
The variation of basic experimental parameters is investigated.
A typical application of the method is discussed and data is
presented for the displacement distribution around an interference
fit fastener.

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ILLUSTRATIONS

SECTION I

INTRODUCTION

Two technical areas which have received considerable attention
in recent years are fracture mechanics and the fatigue performance
of joints utilizing interference fit fasteners. Investigations per-
formed here at the Flight Dynamics Laboratory have required that
measurements be made of the displacement/strain distributions in the
vicinity of cracks and around the edge of fasteners. These data are
used to validate analytical predictions of the strain pattern or to
gain physical insight into structural behavior for the purpose of
formulating new predictive models.

Some of the experimental data were obtained using a relatively
new technique called "speckle diffraction interferometry". This
method which employs lasers and coherent optics is the subject of
this report.

In Section II, the basic phenomena is briefly explained.
Detailed equations have been purposely deferred until later so that
this section can serve as an elementary introduction to the subject
if the reader has not encountered speckle diffraction interferometry
in the past. Section III provides a succinct description of the
apparatus required and how to configure it to perform speckle inter-
ferometry. In Section IV, the speckle phenomena is examined in more
detail. Basic parameters and the effects of their variation are
investigated. The final section characterizes a typical application

of speckle diffraction interferometry. Displacement measurements in the vicinity of an interference fit fastener (tapered bolt) are described. Two appendices have been included to supplement material presented in Section IV.

Throughout this report, the authors have attempted to simplify mathematical descriptions of physical optics phenomena. It has been our experience that those people who have the most need to use this technique are often not familiar with the rather complex mathematical formulations used in physical optics. Many of our arguments are inexact and call upon physical intuition for understanding. This was done, without apology, to encourage a wide spectrum of experimental stress analysis people to consider this new technique.

SECTION II

BASIC PHENOMENA

When using coherent laser light for illumination, a uniform diffuse surface has a speckled or grainy appearance due to random interference within the resolution limit of the eye (or photographic system). The speckled appearance of an object is illustrated by the photographs in Figure 1. It should be noted that the speckle size is dependent on the lens aperture. A magnified view of the speckle pattern is shown in Figure 2.

The speckle phenomenon provides a sensitive method for making measurements of displacements or strains in the surface of an object. Two basic procedures have been developed. The first employs correlation techniques to produce displacement contour fringes within the field of view (References 1, 2 and 3). The second procedure, and the one of interest in this report, exploits light diffraction to measure displacements (References 2 and 4). A double exposure "specklegram" (photo-negative) is made of the speckle surface. Exposures are made before and after a displacement occurs. Thus, each bright "speckle point" is recorded on the specklegram as two dark points - one in the undisplaced position and the other in the displaced position. Only knowledge of the photographic magnification factor is required to determine a local displacement vector once the length and direction of the line segment between corresponding speckle points are known.

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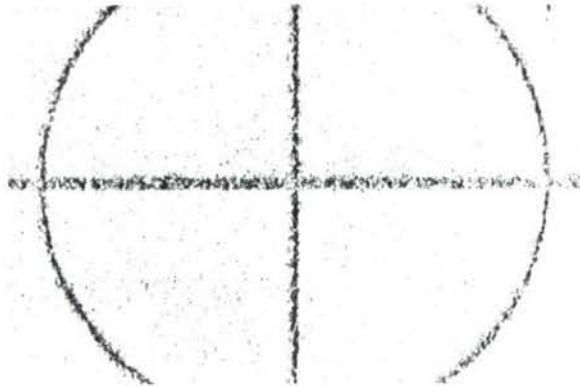
f/8
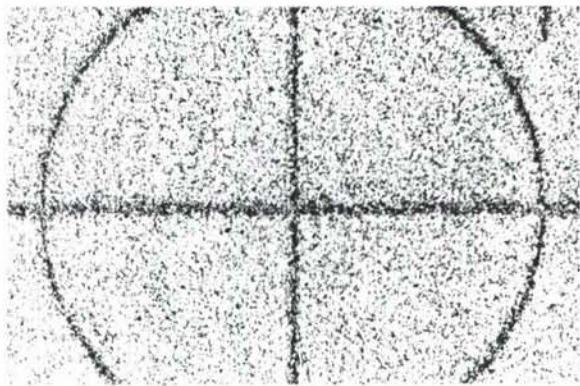
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f/45

Figure 1   Speckled Appearance of Resolution Test
Chart Illuminated with Laser Light
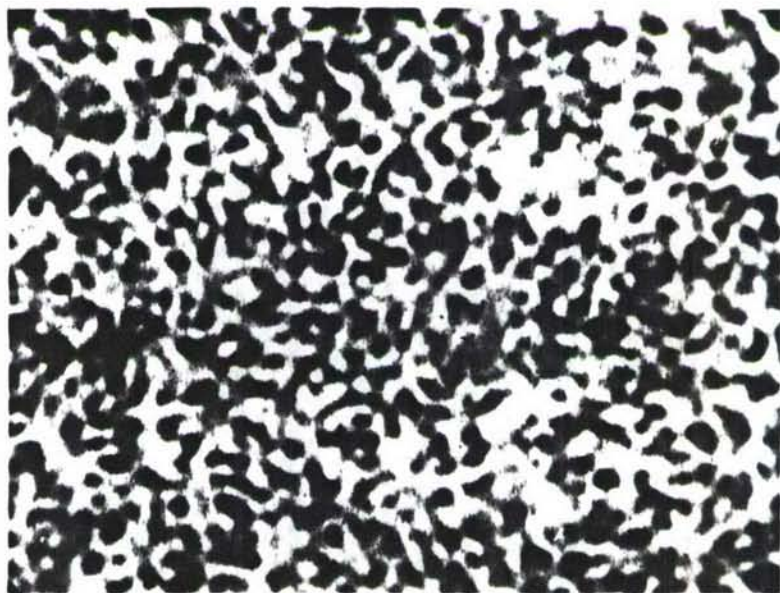
Figure 2   Photograph of Speckle Pattern with
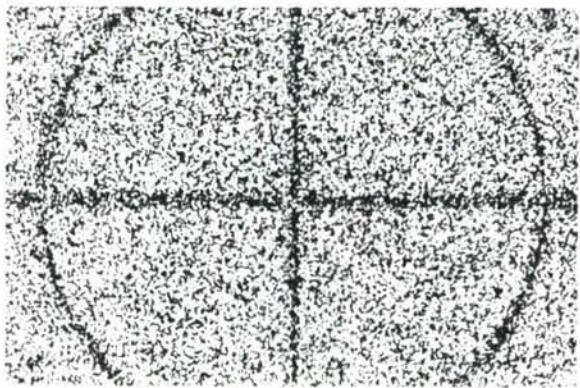300X Magnification

A direct point to point measurement on a specklegram is virtually impossible or at best extremely tedious because of randomness in the speckle field. Fortunately, this is not necessary. If a narrow collimated laser beam (monochromatic and coherent) is directed through the specklegram, diffraction will modify the emerging light rays into a cone. This "diffraction halo" is the result of diffraction from the random distribution of small speckles. Since the speckles are recorded in pairs, however, a parallel fringe pattern (Young's fringes) also occurs in the emerging light cone. Figure 3 is a photograph of a typical speckle diffraction halo with a displacement induced fringe pattern.

For small diffraction angles, Young's fringes are perpendicular to the displacement vector and the spacing is inversely proportional to the displacement magnitude. This provides a measurement of local displacement since, in general, a narrow laser beam is used to interrogate only a small area of the specklegram. The resolution depends on the laser beam diameter and magnification used in the photographic set up. The displacement distribution can be obtained by scanning the laser beam over the entire image area and taking measurements at selected points.

The sensitivity of the speckle diffraction fringe technique for measuring small displacements is comparable to holographic interferometry. It should be noted, however, that this technique is not a substitute, but rather supplements holographic methods. Although in principle, holographic interferometry may be used to measure three

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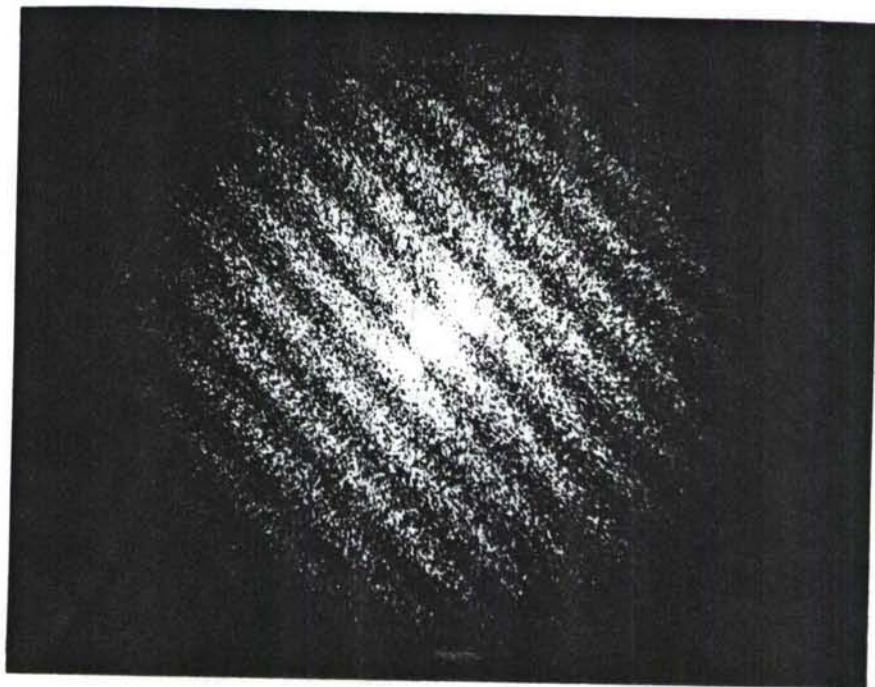
Figure 3   Speckle Diffraction Halo With Young's
Fringes

dimensional displacements, it is not very sensitive for determining

in-plane displacement components. In addition, measurement procedures

are relatively complicated (Reference 5). The speckle diffraction

method measures displacement components which lie in a plane perpen-

dicular to the observation vector. Thus, this technique is

particularly well suited to determining in-plane displacements and

strains. Three dimensional displacements are best resolved by

employing a combination of speckle and holographic techniques.

SECTION III
EXPERIMENTAL APPARATUS AND CONFIGURATION

The experimental arrangement for making a double exposure
specklegram is shown in Figure 4. It is, in fact, a simple camera
configuration. Monochromatic and coherent laser light is used to
illuminate the specimen which provides for a speckle appearance on
a diffuse surface. The lens focal length and object distance are
often selected to yield an enlarged image; however, this is not
always necessary or desirable and depends on the particular measure-
ment to be made. As will be discussed later, the speckles can be
made very small (less than one micron in diameter) so that high
resolution photographic film is normally required. Thin emulsion
holographic films such as Agfa 10E75 are commonly used. At AFFDL,
specklegrams have also been made using Kodak "High Contrast Copy
Film".

Figure 5 is a diagram of the arrangement used to measure
speckle separation. A helium-neon laser with one to fifteen milli-
watts of output will provide adequate illumination. To obtain good
resolution of changes in displacement with respect to location, a
small diameter input beam is preferred (one millimeter is typical).
For a given beam diameter, the spatial resolution is determined by the
selection of an image magnification factor. The fringe pattern is
observed on a screen or through a frosted glass plate. A "main beam
stop" is used to block the undiffracted portion of the laser beam in
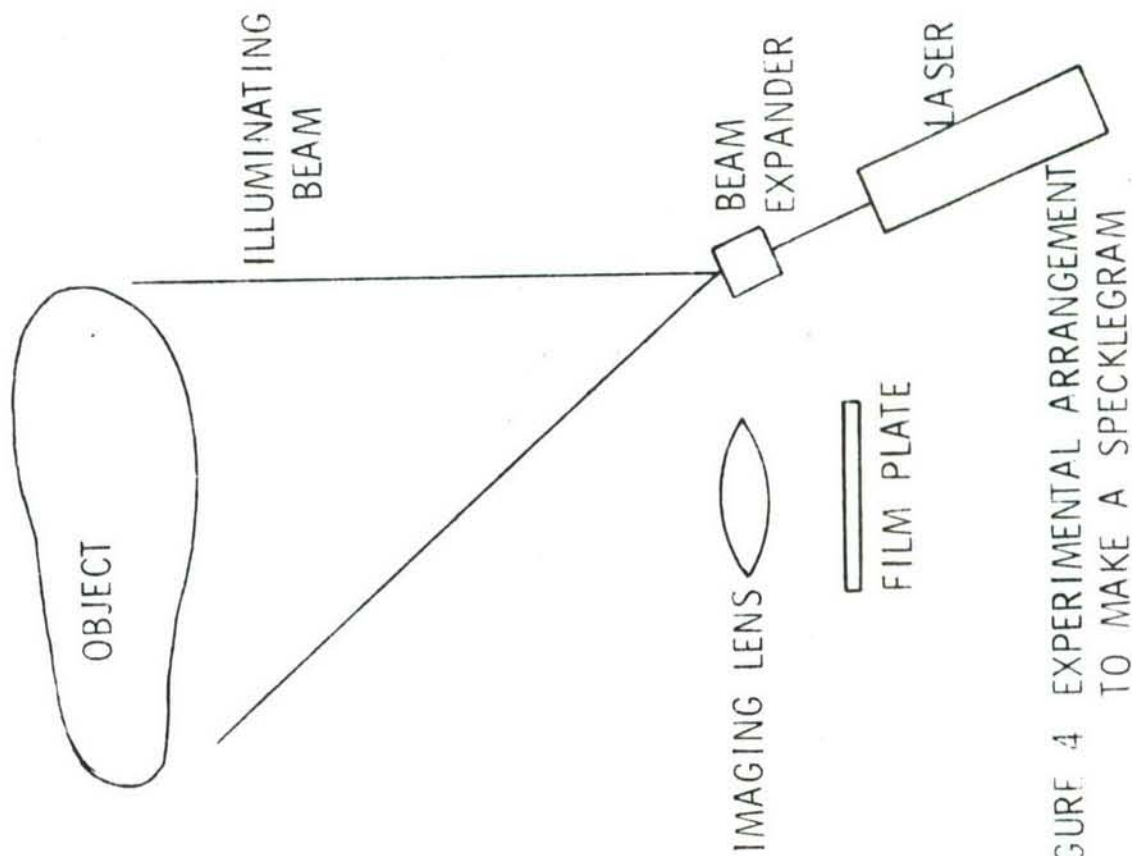
FIGURE 5 EXPERIMENTAL ARRANGEMENT
TO VIEW A SPECKLEGRAM

FIGURE 4 EXPERIMENTAL ARRANGEMENT
TO MAKE A SPECKLEGRAM

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order to obtain maximum contrast. If desired, an optional lens
can be used to collimate the divergent fringe pattern.

SECTION IV

PARAMETER INVESTIGATION

A complete description of the speckle phenomenon requires
knowledge of physical optics and some rather sophisticated mathematics.
Some insight can be obtained, however, using a few simple arguments.
Consider laser light (monochromatic with spatial and temporal co-
herence) to be incident on a diffuse, matte white surface. The
scattered light will have a cosine intensity distribution and a random
phase distribution. This destroys spatial coherence but not temporal
coherence. As a result, interference occurs in the scattered light
field and a random speckle pattern is produced. If no modifying optics
are present, the far field is radially diverging. Thus, by intuition,
one can conjecture that the speckle size in the far field must be
proportional to $\lambda \ell / D$, where λ is the wavelength of light, D
is the diameter of the illuminated area, and ℓ is the distance from
the surface to the point of measurement. The far field approximation
requires that $\ell / D >> 1$. Note also that if the illuminated area
has one dimension greater than the other, the speckles will be
elongated in a direction perpendicular to the long axis. What may be
less obvious is that the constant of proportionality in the speckle
size equation is nearly unity. A formal derivation of this relation-
ship can be obtained by calculating the diffraction pattern resulting
from a continuous distribution of sources of random phase distribution
in an aperture D . Of more interest to this study, however, is the
speckle produced in the image plane of a lens used to photograph the
illuminated surface. This will be discussed next.

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The ultimate purpose of any imaging system is to produce a one to one correspondence between points in the object and image planes. To do this would require a "perfect" lens having an infinite aperture (all light emanating from a point on the object is collected by the lens and focused to the corresponding image point). With coherent illumination, no speckles would be produced (in the image plane) since interference would not be possible with adjacent source points on the object. With any real lens, however, a finite aperture limits the resolution. In other words, an infinitesimal point on the object is mapped into an area in the image plane. The converse is also true; an infinitesimal point in the image plane intercepts light from a finite area in the object plane. From either point of view, it is obvious that interference will take place if the light is temporally coherent and that a speckle pattern will be generated.

The approximate size of a primary speckle cell is just the dimension of an area in the image plane into which an object point has been mapped. An estimate of this parameter can be obtained by computing the "resolution" of the imaging lens. A derivation is given in Appendix A and yields a speckle diameter ℓ given by

$$\ell \approx \lambda F (M + 1)$$

(1)

where M is the image magnification factor and F is the numerical value of the lens aperture for an object located at infinity (i.e., for an $f/4.5$ aperture, $F = 4.5$). Figure 6 contains nine single exposure photomicrographs of speckle patterns in a three by three

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Magnification

1/3   1   3

Lens
Aperture

f/2.8

f/5.6

f/11.
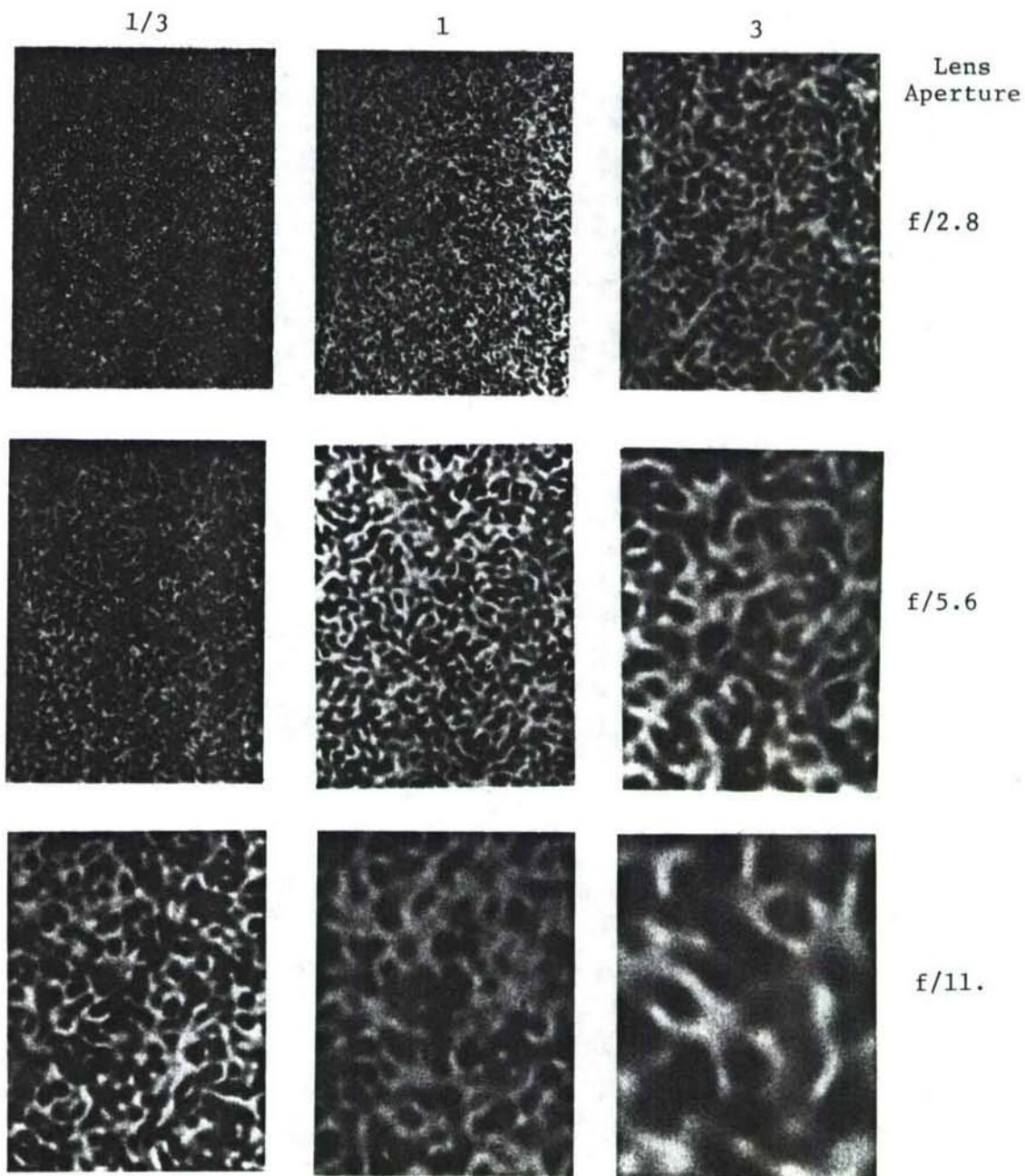
Figure 6   Photomicrographs (150X) which show the
                Variation of Speckle Size with Magnifi-
                cation and f/number

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matrix format. These illustrate the changes in speckle size with variations of magnification (horizontal) and aperture (vertical). These speckle patterns were generated using λ = .6328 microns.

There is of course, some variation in size from speckle to speckle; however, the range is not large. The appearance of larger irregular dark or light areas on the photographs in Figure 5 is due to overlapping of individual primary speckles. This occurs because the spacing between speckles is a random function.

If a narrow laser beam is passed through a transparency of the speckle pattern, part of the transmitted light is diffracted. In practice, most of the diffracted light emerges within well defined angular limits. These angular limits roughly correspond to Airy's disk generated by a speckle size aperture. The far field intensity distribution is a diverging circular cone with an intense center spot from the undiffracted main beam. The center spot is usually eliminated with a main beam stop as is illustrated in Figure 5. The intensity distribution in this diffraction pattern is often called a "diffraction halo". There are, of course, no Young's fringes in the diffraction halo when the speckle pattern is photographed using a single exposure. However, if a double exposure is made with displacements or strain occurring between exposures, then an interferometric phenomenon is observed and the diffraction halo is modulated with Young's fringes. A more detailed discussion of Young's fringes is presented in Appendix B.

Equation (1) will now be used to examine the sensitivity of the speckle diffraction fringe technique for measuring small displacements. It should first be recognized that Young's fringes are readily visible only inside the diffraction halo. Fringe minima have an angular displacement Θ given by

$$(n - \tfrac{1}{2}) \lambda = \Delta_i \sin \Theta$$

$$n = 1, 2, 3 \cdots \tag{2}$$

where Δ_i is the distance between speckle pairs on the double exposure specklegram. From the previous discussion and Appendix A and B, a reasonable approximation for the angular limits of the diffraction halo is given by:

$$\lambda/2 \approx a \sin \beta$$

$$\tag{3}$$

where β is the halo half angle. If for a small displacement, the first minima occur at the edge of the diffraction halo (i.e., $\Theta = \beta$, $n = 1$), then

$$\Delta_i = a$$

$$\tag{4}$$

This indicates that displacements which produce speckle pairs separated by less than one speckle diameter will not generate Young's fringes within the diffraction halo.

Now, a speckle pair separation Δ_i is related to the true displacement on the object Δ by

$$\Delta_i = M\Delta \tag{5}$$

The minimum true displacement that can be measured is, therefore,

$$\Delta_{min} = \lambda/M \tag{6}$$

By substituting Equation (1) for λ into Equation (6), the expression for Δ_{min} becomes

$$\Delta_{min} = \lambda F (1 + 1/M) \tag{7}$$

Equation (7) shows that greatest sensitivity is obtained using a large aperture lens (small F) which is configured to produce a highly magnified image. Note, however, that values of M greater than 3 or 4 do not result in much improvement in the sensitivity. Without the aid of special techniques, values of F much smaller than unity are difficult to obtain. Thus, the ultimate sensitivity appears to be on the order of the wavelength λ . This is comparable to interferometric techniques and also the speckle correlation methods referenced in Section I. One notable advantage to the speckle diffraction fringe method is that it can be desensitized for measuring larger displacements. This is simply achieved by using a smaller aperture with the imaging lens. Holographic and speckle correlation methods are much more difficult to desensitize.

The desensitization of the speckle diffraction fringe method provides a bonus not shared with other interferometric techniques. It is possible to make measurements without a stable platform. Both holographic and speckle correlation fringe techniques require a platform which is vibration free to less than the wavelength of light when continuous wave laser illumination is employed. Our experience has shown that vibration amplitudes up to about 20% of the smallest displacement measured can be tolerated using the speckle diffraction method if the system is appropriately desensitized. It should be noted here that small values of M less than unity will also increase the minimum measurable displacement Δ_{MIN}. This does not really desensitize the technique, however, since the speckle size is nearly unchanging with M if its value is less than unity (compare Equations 1 and 7). A true desensitization is achieved only if the speckle diameter is made large with respect to any extraneous vibration displacements.

One very valuable characteristic of the holographic interferometry and the speckle correlation fringe method is that they provide the observer with a "whole field" view of the displacement distribution. In general, these are contour maps of one component of the displacement distribution. Figure 7 is a photograph taken through a holographic interferogram. The fringes form a contour map of the normal displacement component in a plate subject to a point load. Figure 8 shows speckle correlation fringes on a compact tension specimen. These fringes can be used to measure the horizontal displacement components due to a wedge load.

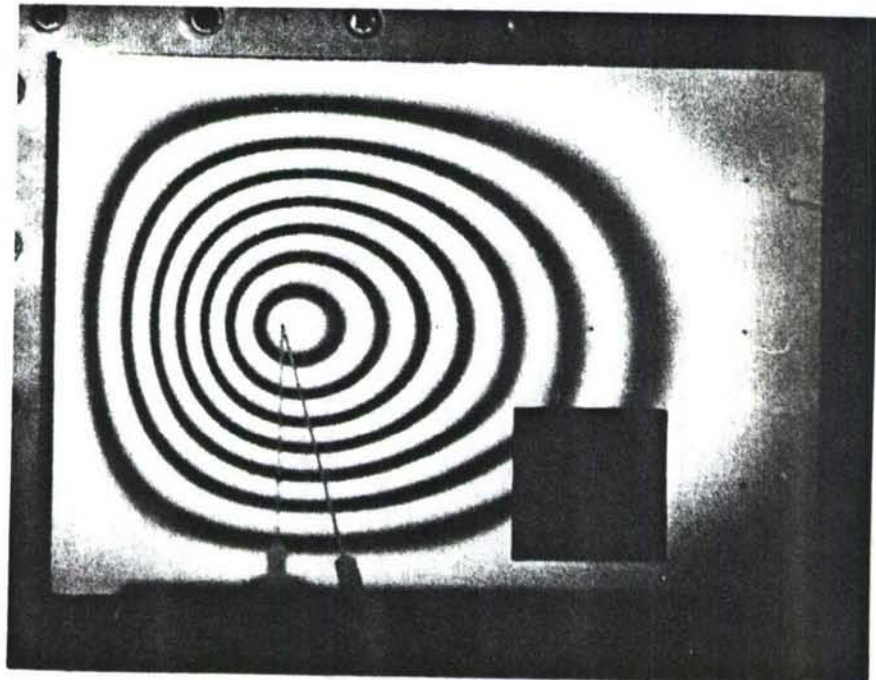P 19

Figure 7   Holographic Fringes on a Rectangular
            Plate Subjected to a Point Load. The
            Plate has a Square "cut out" in the
            Lower Right Quadrant
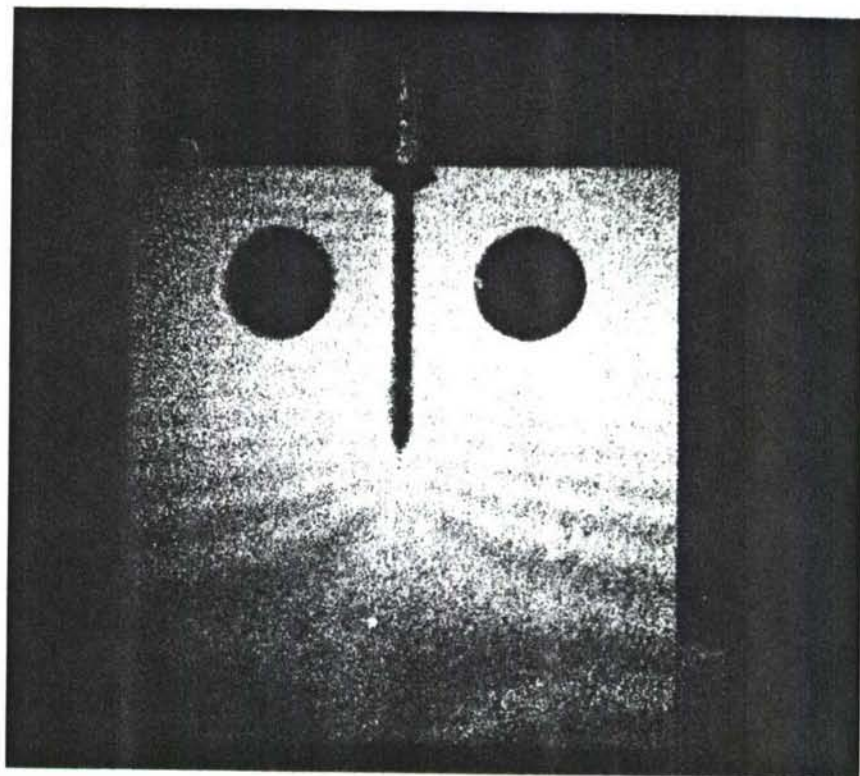
Figure 8   Speckle Correlation Fringes on a Compact
                Tension Specimen

The speckle diffraction technique, as described thus far, provides only a point measurement of the displacement. A distribution is usually obtained by scanning the laser beam over the specklegram image. However, it is possible to generate "whole field" fringe patterns using specklegrams. If the entire specklegram image is illuminated with collimated light, an observer will see diffraction fringes superimposed on the image. Figure 9 is a photograph made using this procedure. These fringes are not contours in the usual sense, which makes interpretation somewhat more difficult. A simple rectilinear coordinate system cannot be used. The displacement component measured has a direction parallel to a line segment ($\vec{V}$) between a point on the fringe and a point on the observation ray. The observation ray is defined by the ray of collimated light passing through the observation point (see Figure 10). The natural coordinates are, therefore, associated with a two dimensional polar system centered on the observation ray. As is true with other interferometric methods, displacement magnitude is proportional to the fringe number.[*]

Quantitative data acquisition using this "read out" technique is accomplished only with difficulty as compared to the point by point method illustrated in Figure 5. However, qualitative and semi-quantitative inspection of a displacement distribution is easily achieved.

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[*]Archbold and his colleagues (References 2 and 4) have demonstrated several other methods for observing the whole field displacement fringe pattern. These techniques are slightly more complicated than the one discussed here. We have not been able to achieve adequate spatial resolution which would allow us to obtain data on a small object when using other than the point read out technique illustrated in Figure 5.

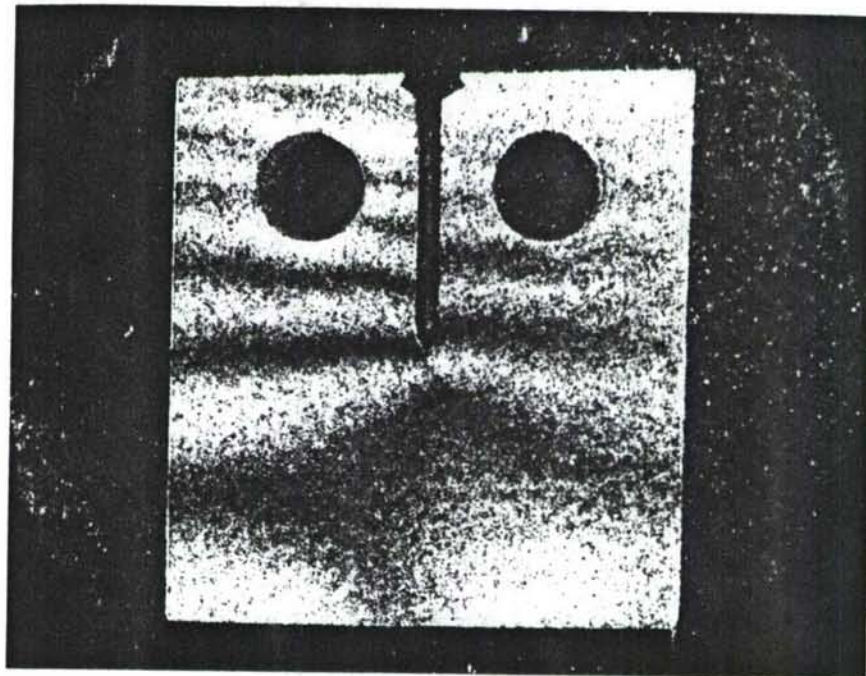Figure 9   Whole Field Speckle Diffraction Fringes
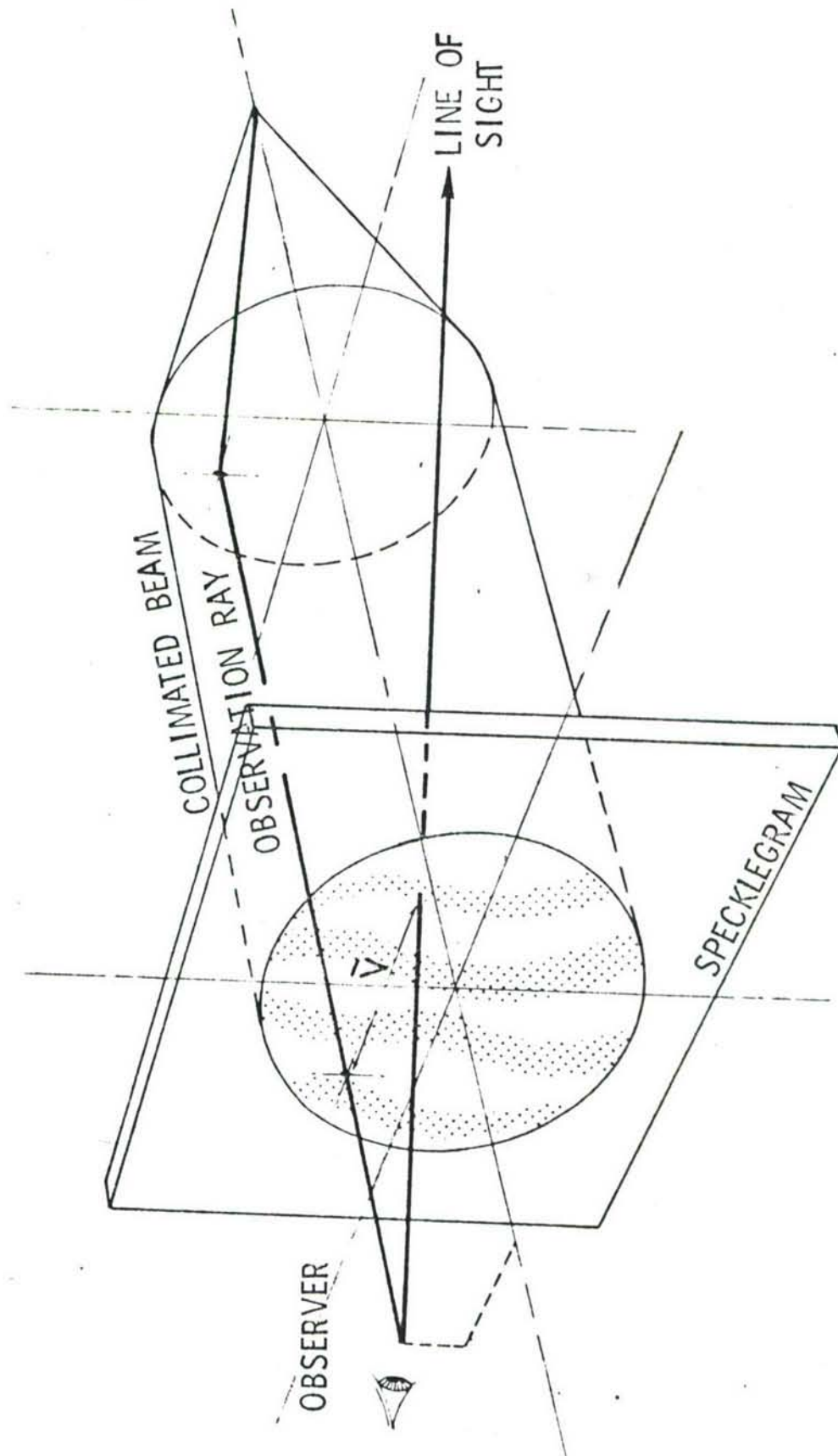   on a Compact Tension Specimen

COLLIMATED BEAM

COLLIMATION RAY

LINE OF SIGHT

OBSERVATION

SPECKLEGRAM

V

OBSERVER

FIGURE 10 VIEWING "WHOLE FIELD" FRINGES THROUGH A SPECKLEGRAM

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SECTION V

A TYPICAL APPLICATION

An interesting and mission relevant problem for the Air Force Flight Dynamics Laboratory concerns the fatigue resistance of structural joints utilizing IFF (Interference Fit Fasteners). These IFF's are employed extensively in advanced aircraft where structural fatigue is critical to the life expectancy of the system. One class of IFF is essentially an oversized tapered bolt which is force-fit into a tapered hole. A one-quarter inch per foot taper is standard for most applications. In a typical installation, a ¼ inch hole diameter is increased by .003 inches when the bolt is forced into place. This usually causes "work hardening" of the material in the immediate vicinity of the bolts and also induces a residual stress pattern. Under certain conditions, both of these results can improve fatigue performance of a structural joint (Reference 6). However, recent experience with IFF installations in new aircraft systems has indicated that the expected improvement in fatigue performance is not always obtained. Although many fatigue failures can be directly traced to faulty installation procedure, or more precisely, inadequate quality control standards, the ordeal has brought to light the fact that the physical mechanisms involved with the employment of IFF's to improve fatigue performance are not fully understood. As a result, the Air Force Flight Dynamics Laboratory has introduced research on a broad front to learn more about this subject. One facet of this

research program involves measuring the strain pattern induced by the installation of an IFF. The "speckle diffraction" technique is a natural candidate for performing this task.

Figure 11 is a photograph of a ¼ inch steel "taper lok" IFF and an aluminum plate containing a tapered hole. In a normal installation, the bolt head is drawn down until it just makes contact with the plate. Since this head would normally shadow the material very near the bolt-plate interface, the head was removed from the IFF (Figure 12). A specklegram was made with exposures taken before and after the IFF installation. A photo-print of the specklegram transparency is shown in Figure 13. The shadow on one side of the bolt is due to the illuminating beam being incident at an angle away from the normal.

Using the techniques described in Section II, displacement measurements were obtained along a horizontal line passing through the bolt center. The results are plotted in Figure 14. Similar measurements were obtained along a vertical line also passing through the bolt center (Figure 15). The data were fit to a curve of the form

$$\Delta = k / r$$

(8)

where k is a "fit" constant and r is the radial distance from the bolt center. Equation (8) is the two dimensional elastic displacement in the vicinity of a circular hole which is loaded radially on its periphery in a uniform manner. The "fit" constant k was determined by the "least-squares" method to be .00012 square inches for both sets of data.
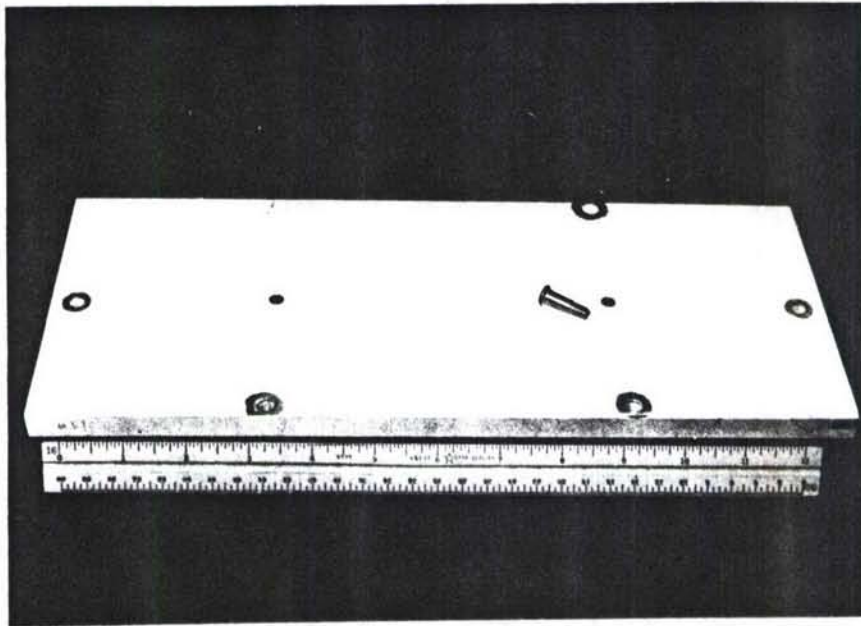
Figure 11   Tapered Bolt Fastener and Aluminum
Plate with a Tapered Hole

Figure 12   Tapered Bolt Interference Fit Fastener.
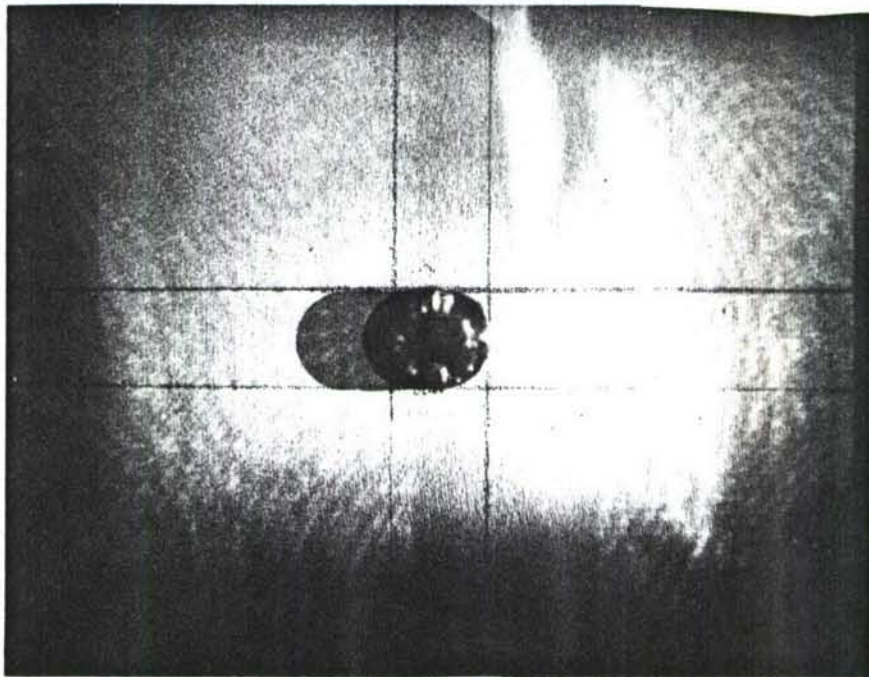The Lower Bolt has the Head Removed.

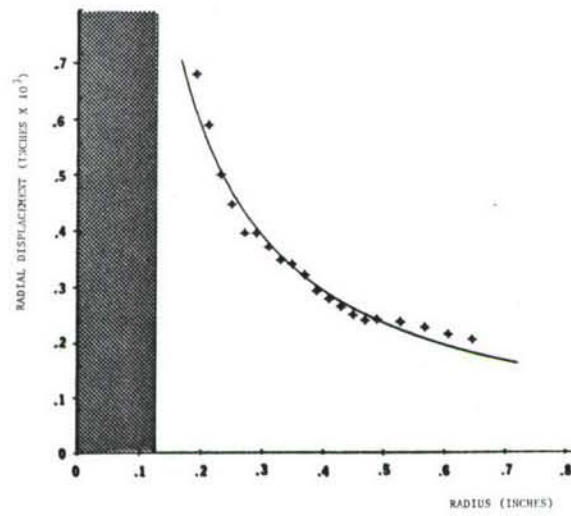Figure 13   Photo Print of Specklegram

Figure 14   Displacement vs Radial Distance Along
a Horizontal Line
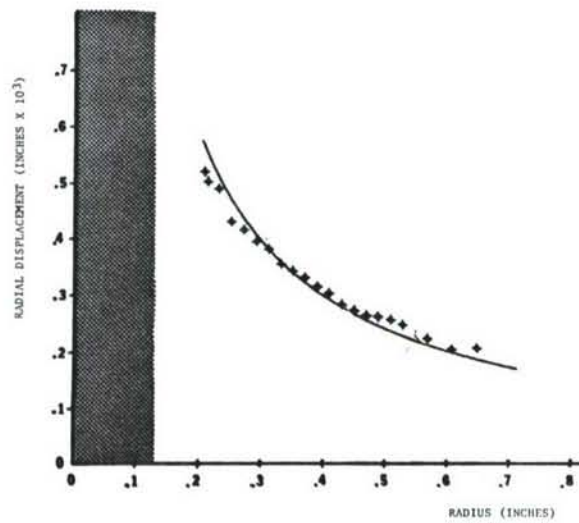
Figure 15   Displacement vs Radial Distance Along
a Vertical Line

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The data plotted in Figures 14 and 15 are for radii greater than .2 inches. No Young's fringes were found when the specklegram was interrogated at lesser radii. The probable explanation for this is that insertion of a tapered bolt causes plastic deformation of the joint material near the edge of the fastener hole. Since plastic deformation is accompanied by rearranging of grain structure, the surface characteristics, and therefore the speckle pattern, are changed substantially. As a result, one should not expect to find speckle "pairs" on a double exposure specklegram in a region that has been plastically deformed.

The above explanation is also consistent with another feature in the data. A value of k = .00012 square inches implies a diametral interference of about .002 inches assuming there is no plastic deformation. The actual fastener was installed to an interference of between .003 and .004 inches. These numbers are based on measurements made of the amount of protrusion found before and after the fastener was installed (the amount of taper is .25 inches per foot). These measurements strongly suggest that plastic deformation did occur near the fastener hole edge. Analytical work done by Allen and Ellis (Reference 7) and others also predicts a plastic zone in the near vicinity of properly installed interference fit fasteners.

Although the same value of the "fit" constant k was found in the horizontal and vertical data scans, some assymetry in the displacement distribution can be detected. In Figure 14, the data appear to fall off slightly faster than k/r while in Figure 15 the opposite is true. This suggests that the hole is not uniformly loaded.

Since the Taper Lok is a precision fastener, manufactured to very close tolerances, the probable cause of non-uniform loading is an out-of-round hole. This explanation is consistent with other observations we have made while experimenting with interference fit fasteners. In particular, assymetric fringe contours have been often observed using "real time" holographic interferometry during the installation of Taper Loks. This has been attributed to out-of-round holes (unpublished work done by the authors at AFFDL).

Before concluding, a few comments about using this technique for measuring displacements in a plastically deformed region are applicable. The disappearance of Young's fringes, as one scans from an elastically strained to a plastically deformed region recorded on a specklegram, is not abrupt. This suggests that very small plastic strains do not completely decorrelate the speckle pattern. In other words, gross changes in the grain structure are not incurred until plastic strains reach some minimum value. As a result, we have been able to use speckle diffraction interferometry within a plastic zone by applying the strain inducing load in small increments. A double exposure specklegram was made for each load increment and the resulting displacement distribution measured. The final displacement distribution can be obtained by superposition (unpublished work done by the authors at AFFDL).

The typical application discussed above demonstrates that speckle diffraction interferometry has a wide potential for measuring displacement and strain distributions in loaded structural components.

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The data presented in Figures 14 and 15 would normally require the installation of more than forty conventional strain gages. To obtain data equivalent to that recorded on the one specklegram shown in Figure 13 would require at least fifty thousand strain gages. In addition, speckle interferometry is rather easy to use and does not require calibration. Other features include: high ultimate sensitivity, variable sensitivity, and the fact that no physical contact is required with the loaded specimen. This last point suggests that speckle technology may be applicable to non-destructive inspection of "strain critical" components.

APPENDIX A

LENS RESOLUTION AND SPECKLE SIZE

Two points on an object can be resolved if the two corresponding
areas into which these points are mapped in the image plane are,
themselves, resolved. The traditional physical optics derivation
involves the Fraunhofer approximation of Kirchoff's diffraction
integral and a calculation of the "Airy disk" diameter (Reference 8).
However, practically the same results can be obtained using a few
simple physical arguments.

The essence of diffraction phenomena involves the fact that
information about spatial distribution is encoded in the diffraction
pattern. For example, if only the zero-order wave from a simple
transmission grating is intercepted, an observer cannot ascertain
information about the grating except for a relative intensity of the
incident illumination. If he gains knowledge about positions of the
first order maxima, he can now relate the slit spacing. The position
and intensity of higher order maxima provide the observer with
information about details concerning the shape of individual slits.
A lens, therefore, can resolve two points on an object only if the
aperture is wide enough to intercept at least the zero order wave
and one of the first order maxima created by diffraction.

Let θ be the angular separation of the zero and first order maxima, and d be the minimum resolvable spacing of two points on the object. Then θ and d are related by

$$\lambda = d \sin \theta$$

<div align="right">(A-1)</div>

If O is the object to lens distance, D the lens aperture diameter and θ is small, the ratio $D/O \approx \sin \theta$ so that Equation (A-1) can be written

$$\lambda = d\, D/O$$

<div align="right">(A-2)</div>

In the image plane, the spacing should be

$$d_i = M d$$

<div align="right">(A-3)</div>

where M is a magnification for an image plane located at a distance I behind the lens. From simple geometrical optics come equations

$$M = I/O$$

<div align="right">(A-4)</div>

and

<div align="center">34</div>

$$\frac{1}{f} = \frac{1}{I} + \frac{1}{O}$$

<div align="right">(A-5)</div>

where f is the focal length of the lens.

Equation (A-3) can be rewritten in terms of Equations (A-2), (A-4) and (A-5) and become

$$d_i \approx \lambda (M+1) f/D$$

<div align="right">(A-6)</div>

Note that f/D is just the numerical value of the lens aperture for an object located at infinity and can be denoted by F (i.e., for an $f/4.5$ aperture, $F = 4.5$). Previous arguments indicated that d_i is a reasonable estimate of the speckle diameter which will now be denoted by λ. Thus, Equation (A-1) can be written as

$$\lambda \approx \lambda F (M+1)$$

<div align="right">(A-7)</div>

A formal derivation of Equation (A-6) yields a multiplicative constant of 1.2. In view of the approximate nature of other assumptions, the difference between 1.2 and unity is not significant.

APPENDIX B

YOUNG'S FRINGES

"Scalar Wave Theory" provides that the far field intensity
distribution of diffracted light is closely related to the Fourier
transformation integral (Reference 8). In particular, the intensity
I is proportional to a scalar product UU^* where

$$U \approx \int_W dy \, dx \, \frac{1}{S} e^{ika}$$

(B-1)

Equation (B-1) represents a summation over an aperture W contained
by the x-y plane at z = 0. The wave number $k = 2\pi/\lambda$ and S is
the distance from a point in the aperture to the point of observation.

Now consider an aperture consisting of two small pinholes
located at $x = \pm \Delta/2$, $y = 0$ where Δ is the pinhole
spacing. The pinholes can be mathematically represented by two
dimensional Dirac delta functions. A first order approximation for
the distance variation is $S = S_o + x \sin \theta$ where S_o is
the distance from the origin to the observation point and θ is the
angle this line segment projects in the x-z plane as measured from the
z axis. This approximation is valid when the distance from the
observation point to the z axis is small compared to S_o. With this
aperture, Equation (B-1) can be written

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$$U \approx \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dx \; \frac{1}{S_o + x \sin \theta} \; e^{ik(S_o + x \sin \theta)} \left[\delta_{(x-\frac{\Delta}{2}, y)} + \delta_{(x+\frac{\Delta}{2}, y)} \right]$$

(B-2)

If Δ is very small compared to S_o, the quantity $x \sin \theta$ can be neglected in the $1/(S_o + x \sin \theta)$ factor. Thus:

$$U \approx \frac{e^{ikS_o}}{S_o} \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dx \; e^{ikx \sin \theta} \left[\delta_{(x-\frac{\Delta}{2}, y)} + \delta_{(x+\frac{\Delta}{2}, y)} \right]$$

(B-3)

or

$$U \approx 2 \frac{e^{ikS_o}}{S_o} \cos\left(k \frac{\Delta}{2} \sin \theta\right)$$

(B-4)

By employing $I \propto U U^*$, the intensity function for Young's fringes is obtained and is

$$I \propto 1 + \cos(k \Delta \sin \theta)$$

(B-5)

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By using $k = 2\pi/\lambda$ and noting that minima occur for $\cos(k\Delta\sin\theta) = -1$ the conditions for fringe minima are

$$(n - \tfrac{1}{2})\lambda = \Delta\sin\theta$$

$$n = 1, 2, 3 \cdots$$

$$(B-6)$$

Now consider a case where the aperture contains a speckle pattern. Let $R_{(x,y)}$ be a distribution function describing the spatial character of the speckle pattern. The diffraction integral is then

$$U_h \approx \frac{1}{S_o} \int_W dy\, dx\, e^{ik\, S_{(x,y)}} R_{(x,y)}$$

$$(B-7)$$

The function $U_h U_h^*$ as calculated from (B-7) is the diffraction halo for a single exposure speckle pattern as described in Section II. It follows that the U function for a double exposure specklegram can be written as

$$U \approx \frac{1}{S_o} \int_W dy\, dx\, e^{ik\, S_{(x,y)}} \left[R_{(x-\frac{\Delta}{2}, y)} + R_{(x+\frac{\Delta}{2}, y)} \right]$$

$$(B-8)$$

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It is now useful to use a series expansion

$$R_{(x \pm \frac{\Delta}{2}, y)} = R_{(x,y)} \pm R'_{(x,y)} \left(\frac{\Delta}{2}\right) + \frac{R''_{(x,y)}}{2!} \left(\frac{\Delta}{2}\right)^2 \pm \cdots$$

(B-9)

where primes indicate a differential with respect to x. Also, an appropriate linear first order approximation for $S_{(x,y)}$ is $S_0 + V_{(y)} + x \sin \theta$ where $V_{(y)}$ carries the functional dependence on y. Now Equation (B-8) can be written

$$U \approx \frac{e^{ikS_0}}{S_0} \int_{-\infty}^{\infty} dy \, e^{ikV_{(y)}} \int_{-\infty}^{\infty} dx \, e^{ikx \sin \theta} \, \Phi_{(x,y)}$$

(B-10)

where

$$\Phi_{(x,y)} = 2\left[R_{(x,y)} + \frac{R''_{(x,y)}}{2!} \left(\frac{\Delta}{2}\right)^2 + \frac{R''''_{(x,y)}}{4!} \left(\frac{\Delta}{2}\right)^4 + \cdots \right]$$

(B-11)

The integration limits have been set at $\pm \infty$ since it is assumed that $R_{(x,y)} = 0$ outside the laser beam. Now, the second integral in Equation (B-10) is essentially a Fourier transformation with a spatial variable of x and a frequency variable of $-k \sin \theta$. Hence, apart from a constant the Fourier transform is:

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$$\mathcal{F}_{(\omega,y)}\left\{\Phi_{(x,y)}\right\} = \int_{-\infty}^{\infty} dx\, e^{-i\omega x}\, \phi_{(x,y)}$$

where $\omega = -k \sin\theta$

The linear properties of a Fourier transform provide that

$$\mathcal{F}_{(\omega)}\left\{g_{(x)}^{(n)}\right\} = (i\omega)^{n}\, \mathcal{F}_{(\omega)}\left\{g_{(x)}\right\}$$

where $g_{(x)}^{(n)}$ is the (n)th derivative of a function $g_{(x)}$. Thus, by using Equation (B-11) and (B-13), Equation (B-12) becomes

$$\mathcal{F}_{(\omega,y)}\left\{\Phi_{(x,y)}\right\} = 2\mathcal{F}_{(\omega,y)}\left\{R_{(x,y)}\right\}\left[1 - \frac{1}{2!}\left(\omega\frac{\Delta}{2}\right)^{2} + \frac{1}{4!}\left(\omega\frac{\Delta}{2}\right)^{4} - \cdots\right]$$

or

$$\mathcal{F}_{(\omega,y)}\left\{\Phi_{(x,y)}\right\} = 2\mathcal{F}_{(\omega,y)}\left\{R_{(x,y)}\right\}\left[\cos\left(k\frac{\Delta}{2}\sin\theta\right)\right]$$

If Equation (B-15) is substituted for the second integral in
Equation (B-10), it should be recognized that apart from the factor
$2 \cos \left(k \frac{\Delta}{2} \sin \theta \right),$ Equation (B-10) and Equation (B-7) are the
same. That is

$$U = U_h \, 2 \cos \left(k \frac{\Delta}{2} \sin \theta \right)$$

(B-16)

and the intensity is

$$I \propto U_h U_h^* \left[1 + \cos \left(k \Delta \sin \theta \right) \right]$$

(B-17)

Thus, the diffraction pattern from a double exposure specklegram is
a diffraction halo modulated with Young's fringes which are intimately
related to the displacement Δ by Equation (B-6).

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13 ABSTRACT

The procedure for using laser speckle diffraction interferometry to measure
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mental parameters is investigated. A typical application of the method is
discussed and data is presented for the displacement distribution around an
interference fit fastener.

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